

Partial Prestressing of Concrete

A Review of Design Data and Requirements

S. Kryzevicius
Project Research Engineer
Structural Research Group
Research and Development Branch

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Abstract: The purpose of this report is to supplement the "State-of-Art Report on Partially Prestressed Concrete Bridges" prepared by Queen's University for the Ministry of Transportation and Communications (MTC) in 1983. At the same time this report is intended to collect the relevant data for the pending MTC publication by the same author "Proposed Design Provisions for Partially Prestressed Concrete Bridges".			
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1/ INTRODUCTION

The "State-of-the-Art Report on Partially Prestressed Concrete Bridge Structures" [2] was prepared in 1983 for the Ministry of Transportation and Communications (MTC) by Queen's University Professor B. deV. Batchelor and Research Assistant M. El-Shahawi. The report provides consolidated technical information available at this time in the English language.

The purpose of the review herein is to provide a commentary on the state-of-the-art report [2] and to supplement the information given therein. It is advisable to read the state-of-the-art report [2] prior to reading this review.

Both of these presentations, attempt to establish the relevant data needed by both the Code writers and design engineers.

2/ REVIEW OF REFERENCES

A limited number of articles and books refer directly to the subject of partially prestressed concrete. Subjects related to criteria like crack control or fatigue of reinforcing steel are mostly discussed with respect to reinforced or fully prestressed concrete. ACI and PCI Journals may be considered the main sources of information on the subject of partial prestressing available in North America.

Bulletin #479 of the University of Illinois, published in 1965, contains an extensive list of references on the subject of crack control of concrete [1]. Today, this publication has historical value only. The preliminary state-of-the-art report by Queen's University [2] is the first publication in Canada which provides a selection of information on the method of partial prestressing in bridges.

P.W. Abeles may be credited with the introduction of the subject of partial prestressing in England and the U.S., circa 1945 [3]. B. Thurlimann introduced the method to Continental Europe, about 1968 [38]. Since then, partial prestressing has become popular in Switzerland and it has almost entirely replaced the methods of full and limited prestressing [41]. Some references to German technical publications will be provided in order to illustrate Swiss and German experience in this type of design.

One of the best known articles on the subject of crack control in Central Europe is the article by G. Rehm and H. Martin [4] published in 1968. Several of the methods of crack control currently used in Europe are based on the original formulations by these authors [6 and 7]. The FIP-CEB committee [5] have adopted equations which have been derived from original equations in Reference 4. References 8 and 9 represent recent efforts by German engineers to readjust these equations for use in partially prestressed concrete thereby accommodating findings of recent tests intended to establish the effect of bond on crack formation in post-tensioned members. References 10 to 17 discuss the subject of crack control published in the English language. Of these, Reference 16 deserves the reader's attention as it evaluates the crack control equation adopted by the Ontario Highway Bridge Design Code (OHBDC) [29].

For information on fatigue-related cracking in concrete and fatigue resistance of reinforcing steel, refer to References 18 to 27. Among them, Reference 25 lists over 100 sources of information related to fatigue in reinforced, fully prestressed, and partially prestressed concrete beams. The ACI publications [19 and 22] contain fatigue test data for reinforcing and prestressing steels of different origins or methods of fabrication.

Probably the most comprehensive treatment of serviceability and ductility criteria in concrete is given by F. Leonhardt in References 6 and 27. In comparison to the above sources, the information provided in the references concerning deformation and ductility criteria in the Queen's University report [2] appears to be less convincing. Of these, only Reference 31 is an exception.

The flexural analysis is, as usual, one of the major topics of engineering discussions. The articles in References 28 to 38 have been reviewed and selected from the references found in the Queen's University report [2]. References 41, 42, and 43 are added to these in order to give a representation of the design trends of partially prestressed concrete followed in Switzerland, and Reference 27 has been selected to represent those followed in Germany. It will be important to note that References 30 and 31 give information for the analysis of prestressed beams at the ultimate states having full and partial degrees of prestress, respectively. Comparisons may prove that no significant difference exists in these two methods.

Interesting observations have been made by M. Herzog in Reference 43. When comparing the spectrum of 597 flexure tests on concrete beams, using an identical analysis pattern and one set of equations for all tests, he concluded that the pattern of analysis at ultimate state for reinforced concrete may be suitable for partially and fully prestressed concrete, as well. In these tests of 69 partially prestressed beams, nine with unbonded tendons were evaluated. Among the samples tested many were of U.S. origin. M. Herzog in [43] and F. Leonhardt in [27] voice the opinion that Bernoulli's hypothesis stating that "plane forces remaining plane" can often be questioned at the level of ultimate failure load. The strain diagrams at ultimate failure load become non-linear [27].

References 27 and 42 provide information on shear treatment in partially prestressed beams. This important subject has been given little attention in the past.

3/ APPLICABLE DESIGN CRITERIA

The selection of the prestress level may depend on the degree of exposure to severe environmental elements.

In analogy to the 1970 CEB-FIP [15], the Queen's University report [2] has proposed four concrete classifications relating to four levels of prestressing. Crack formation in concrete at service loads is not permitted in Classes I and II. Partially prestressed concrete has been assigned to the Class III prestressing level. Ordinary reinforced concrete was designated as Class IV concrete. The logistics of this classification suggest continuity of common criteria in all the above four classes of concrete.

The following has been proposed in the Queen's University report [2] for the limit states criteria.

Serviceability:

- crack control of concrete,
- load repetitions and fatigue, and
- deflection or deformation.

Ultimate limits:

- strength, and
- ductility and compatibility.

In the following pages a brief summary will be provided to illustrate major findings on these subjects as stated in the Queen's University report [2]. Pertinent additional information, much of it new, will be added from other sources to encourage further discussion.

4/ CRACK CONTROL OF CONCRETE

Reference 2 suggests that a large discrepancy in the results is to be expected from the selected equations predicting crack widths. Further study of this subject is suggested by this review and the following information should be included.

F. Leonhardt [6] indicates that in partial prestressing some essential changes in crack formation pattern are to be expected.

The general consensus in the engineering profession is that none of the well-known crack width formulae will be adequate when applied to partial prestressing [35]. Although some of these are often ignored in the experimental equations, the established general formulation of the crack widths equations should include the parameters as listed below [4, 6]:

- Effect of concrete cover and spacing of reinforcing bars
- Bonding properties of steel
- Type of loading -- tension, flexure, shear, torsion etc.
- Load repetitions and fatigue including dynamic effect

Of these parameters, bond and fatigue deserve special attention. Often the bonding properties assumed in the available equations do not represent true conditions for post-tensioned members. Although mixing different types of reinforcing and prestressing steels, with their respective differences in bonding conditions, has been attempted, most crack control equations assume direct contact of prestressing steel with the concrete [2]. It is apparent that such a situation should be evaluated further.

The bonding properties of wire strands in pretensioned members compare favourably with the bonding assumed for deformed bars. Different bonding conditions have been found to exist in post-tensioned tendons which are placed and pressure-grouted within the sheaths. The investigation by V. Thormählen [8] has established that bond values should be reduced for the tendons grouted within the deformed sheaths. In comparison with bond values of deformed bars embedded in concrete, he proposes 80% bond value reduction for smooth prestressing steel, 60% bond value reduction for

wire strands, and 36% reduction for prestressing bars with ribs of external deformation geometry.

These findings [8] are similar to the proposals in Reference 27, which recognizes different characteristics of bond as established by H. Trost in a series of recent tests.

The original crack width equation by Rehm and Martin has been rewritten by Thormählen to accommodate partially prestressed sections which may be either post-tensioned or pretensioned. Equation 10, as proposed by Thormählen has been verified by testing of actual performance in 18 tension members. Tests reported in [27] record nine specimens tested by Trost. The pull-out tests were done in grout of 30 MPa strength for reference slips of $\Delta = 0.1$ mm and $\Delta = 0.5$ mm, respectively.

Crack control during repeated application of load is a feature which is seldom accommodated by the available equations. Only References 24 and 25 of the Queen's University report [2] list CEB-FIP Equations 2.8 and 2.10, and Equation 2.17 by Dean and Park which accommodate fatigue features. For eventual selection of the proper crack control equations, designers should realize the importance of bond and the factor of load repetitions discussed. So far, only Equation 10 by Thormählen [8] has all the previously listed features. It is apparent that in the case of pretensioned sections with equal steel bonding properties, selection of formulae is less restricted. The CEB-FIP equations, as noted above, appear to be the most suitable.

It is essential that the above-discussed equations by Thormählen be simplified for everyday design purposes. This has been done in Reference 9. Eventual graphical presentation would be most suitable. F. Leonhardt [6], provides many useful design aids, among which he recommends that certain limits of crack widths and spacing not be exceeded.

These recommendations are mainly for reinforced concrete and provide the maximum limits for bar spacing. In partial prestressing, the $\Delta\sigma_a^{II}$ and τ would represent the relative increase of stresses in reinforcing and prestressing steels ($A_v + A_s$) above the full decompression level (see Chapter 8). These values for maximum bar spacing are provided in

the following table for critical width of crack with 90% probability of cracking:

Table 1/ Crack Widths and Maximum Bar Spacing

Width of crack $W_{90\%}$	0.1	0.2	0.4 mm
Tension member	100	150	200 mm
Flexure for $\Delta\sigma_a^{II} = 240$ MPa tension	100	150	200 mm
Flexure for $\Delta\sigma_a^{II} = 120$ MPa tension	150	200	300 mm
Shear stress in vertical stirrups $\tau_0 = 3$ MPa	50	100	150 mm
Torsion stress in vertical stirrups $\tau_T = 2$ MPa	50	80	150 mm

5/ FATIGUE CRITERIA

Load repetitions affect the pattern of crack formations by destroying part of the bond between reinforcing steel and concrete adjacent to cracks. In partially prestressed concrete this situation may be considered to be equal or less critical than in reinforced concrete mainly due to action of compressive force. However opinions differ. Usually, when checking the fatigue criteria the design analysis specifies that the fatigue endurance of steel be ensured. This would be satisfied by allowing the stress range up to the certain specified limits. The check should apply to reinforcing steel, prestressing steel and the prestress anchorages. In the partially prestressed beams after concrete crack formations, stress range in steel gradually increases and often can reach full reversal of stress; this performance is identical to that of ordinary reinforced beams. Leonhardt [27] anticipates that the stress range should not exceed 150 MPa in partially prestressed beams. As stress reversal into tension never happens in fully prestressed beams, stress range in fully prestressed beams should usually remain below 85 MPa.

Conclusions drawn in the Queen's University report [2] are based on observations made by Rao and Babu [21]. Their Figures 3.1, 3.2 and 3.3 indicate the anticipated stress ranges which are informative and of great interest. The report then refers to the check of reinforcing steel which is based on the ACI publications by Committees 215 and 443 [19, 45]. Reinforcing steel data compares with findings in Reference 23 found in the OHBDC, Section 11 [29]. But all this information is still inadequate for partially prestressed concrete.

Design information for evaluating the performance of prestressing steel separately from the anchorages and the couplers is particularly difficult to find. This information has, however, been published by German researchers in References 24, 27 and 46, which provide guidelines for fatigue and brittle fracture treatment in several types of prestressing anchorages. The articles in References 19, 22, 26 and 27 provide summaries of tests and pertinent considerations for fatigue performance of prestressing steels (including those in partially prestressed concrete). The articles in References 25, 32, 34 and 37 provide proposals for fatigue oriented design analysis. The authors of these latter

references provide, each in his own way, proposals which attempt to resolve the fatigue investigations.

In the Appendix, permissible stress ranges from [27] are quoted for anchorages and prestressing steels manufactured in Germany.

Several conclusions will be drawn from the above. Some of these are listed below and are presented for the purposes of discussion in the light of the OHBDC, Section 11 requirements [29].

- Concrete exhibits a lower strength value under frequently repeated loads and the concrete apparently does not have a set value of fatigue endurance limits [25]. Cracking of concrete does not relate to the specific number of load applications [4, 6, 19 and 25]. The rate of load application, the eccentricity of load, and the given reinforcing index are major parameters to be considered. The tensile and compressive strength, bond, and shear are affected.
- The stress range parameter shall be used for the evaluation analysis of reinforcing and prestressing steels or concrete. This approach should apply to all four concrete classes. However, due to lack of tensile stress, the stress range in fully prestressed sections should seldom be critical. Exceptions may be made to the anchorages and couplers in the analogy to DIN 4227 Clause 15.9 [44].
- Design rules established for reinforced concrete and fully prestressed concrete shall apply to partially prestressed concrete. However they do relate to the degree of prestressing. Suggestions for partial prestressing found in [27] and [41] are given in the Appendix.
- Section 11 of the OHBDC [29] may be applied for fatigue evaluation in partially prestressed concrete.

The subsequent comments related to fatigue criteria are gathered from these sources: [19, 21, 22, 24, 25 and 26]. Steels of foreign origin have properties different to those manufactured in the U.S. or Canada. In addition, interpretation of test data may be oriented towards

different engineering and safety standards. Despite these factors, the information provided may be highly informative and, with some exceptions, these foreign steels are available on the Canadian market [19].

In References 19, 24 and 44, data are given for permissible stress ranges in anchorages and couplers. For a fatigue endurance limit of 2×10^6 load application cycles, stress range shall not exceed the permissible stress range of 75 MPa. For clamp-type or wedge-type anchorages stress range should be reduced even further. The tests indicate that the lowest values of fatigue resistance have been established in threaded clamp anchorages. Loop-type anchorages should allow only 80% to 50% of permissible stress range assigned to wires or strands outside the stress transfer areas ([44] p. 304, Clause 15.9, Volume 1).

The prestressing steels are usually tested for fatigue without steel being embedded into concrete. Several tests reported in [22] and [25] indicate a drop in the endurance limits of steel (after concrete cracking) if steel was embedded. This phenomenon was common for prestressed and non-prestressed steels.

The comparable endurance limits specified for fatigue in ordinary reinforcing and prestressing steel are expected to be of higher order if the investigation relates to prestressing steels. This assumption, however, is not always true. The provisions for wires, strands and bars listed in [19] and [29] for steel types marketed in Canada indicate, for example, the permissible stress range specified for high-strength reinforcing bars by the OHBDC [29] is 125 MPa. For bends and welded reinforcement a further 50% reduction applies. For prestressing bars in [19] the limit is $0.1 \times f_{pu} = 103$ MPa. These limits for stress ranges are much more restricting than those listed by DIN below.

It is of interest to note that DIN 1045 [44] in Table 6.6, page 134, Vol. II, specifies a permissible stress range of about 200 MPa for reinforcing bars and 120 MPa for welded bars, respectively. The reason for such a large discrepancy in provisions is not known but it should be indicated that the actual stress ranges anticipated in steel of partially prestressed beams will often exceed the 120 MPa value.

The article in [26] provides further fatigue test data for several prestressing steels used in Germany. The tests indicate that stress range endurance limits vary between 200 MPa and 600 MPa. The smooth bars and smooth wires typically present higher levels of endurance. The lowest endurance values have been established for hot-rolled wires and strands. Cold-drawn wires, wires with stabilized stress, or cold-drawn bars indicate improved performance. The surface deformations on bar or wire steels tend to create a notch effect. Due to danger of brittle fracture, 20% to 50% permissible stress range reduction is suggested for these steels in [26]. The specified reduction in [19] is even higher.

In [26] stress range effect was checked for two upper stress levels. The $\sigma_0 = 0.55 \beta_N$ and $\sigma_0 = 0.9 \beta_{0.2N}$, respectively. Where β_N is the tensile strength of steel and $\beta_{0.2N}$ is 0.2% proof stress of steel, respectively. The number of single amplitude stress cycles was $N=2 \times 10^6$. Both σ_0 stress levels were prescribed by German Code, the second of which was simulating the bends.

The steel stress corrosion was not reported in [26] but this phenomenon is well known and it was discussed in [46]. The steel corrosion fatigue tests reported in the article [26] do not indicate that conclusive results could be reached at that time. Reduction of endurance as reported from two over-conservative tests was between 36% and 79%. Further research of corrosion-related problems is indicated.

Friction corrosion was investigated in sections prestressed with wires and wire strands (after crack formation in the concrete). No specific reduction value has been provided [26]. Such friction corrosion phenomena have been reported in other tests [19, 25 and 27].

The article [26] indicates 10% to 20% reduction of endurance in zinc galvanized wires and strands.

The brittle fracture phenomenon has been investigated for temperatures between +20°C and -100°C and between +20°C and -50°C, respectively. The tests were made on smooth wires and prestressing wire strands. These tests were intended to complement the brittle fracture data for deformed

bars and wires which was known beforehand. Conclusions were not indicated. Sources [6] and [25] indicate increased danger in brittle fracture of deformed steels. Steel types sensitive to "notch" effect or stress concentrations due to deformations can be found in [19] and [26].

Some further discussion on the fatigue performance is found in [41] and may be considered. In these discussions, Dr H. Bachman indicates that so far in all the tests for fatigue, the failures in partial prestressed concrete tended to occur in non-stressed ordinary reinforcing prior to those in prestressed tendons. This indicates that after concrete cracking the stress increase in reinforcing and prestressing steels is not equal.

The pattern of predictions of failures in couplers and prestress anchorages at the end supports should be identical to those in fully prestressed beams. However, a check only applies to fatigue critical areas like intermediate anchorages or couplers. These should be avoided in partial prestressing.

The fatigue should not be considered only as specific phenomena for partial prestressing. On the contrary, this criteria often is more favourable than in reinforced concrete. It has been established that the fatigue is seldom critical in fully prestressed beams with bonded tendons.

Concluding, the ACI provisions prepared by Committee 443 appear to be more conservative than may be suggested by German or Japanese tests as found in the above-mentioned references. A critical evaluation of the currently used fatigue provision is essential since the actual stress range of partially prestressed girders may exceed 120 MPa as indicated in References 27 and 41.

6/ DEFLECTION OR DEFORMATION

In bridges, deflection criteria are seldom critical for partial prestressing. The writers in [2] are of the opinion that the deflection criteria are adequately covered in the available literature. Then they suggest that measured deflections of partially prestressed concrete as reported are usually within the limits specified by local codes. Analysis involving continuity, moment redistribution, and long-term deflections were not clarified.

However, extensive discussions provided in [6] do not present the situation discussed above in such a simple light. The following points out that at the serviceability level, ignoring concrete participation between the cracks plus eliminating the reinforcement index from these investigations, or by further over-simplifying the creep factors, the designer may be working with inaccurate information and thus should not expect to obtain accurate results. To illustrate part of this problem, Reference 43 suggests the correction factor for tensile participation of concrete between cracks reaches 17%. Secondly, at loads close to failure of post-tensioned sections, a loss of bond and nonlinearity of strains has been indicated in [27] and [43].

It is hoped that the ideas and suggestions provided by Leonhardt will be adequate to stimulate further investigations of this particular situation. For the time being, however, the information provided in [2] may be applied, where the approximate deflection or deformation values are acceptable.

7/ DUCTILITY

This criteria has been selected in [2] to be a separate item in the ultimate limit states. The purpose of such a division is not clear. The report [2] bases the findings on the investigations by Cohn and Bartlett in its Reference 83, and in Reference 52 of Thomson and Park. Then the report concludes that ductility of partially prestressed beams is dependent on the following:

- content of prestressing steel and the distribution of same within the section; and
- percentage and type of transverse reinforcement and the effect of concrete cover.

The parameters of mixed reinforcement index γ , partial prestressing index K_0 , and net reinforcement index ω have been chosen as the most dominant factors.

This information provided is important and may be complementary to the ductility information as provided in [6, 41, 42 and 43]. Some of this may be discussed in Section 8 when presenting the opinions expressed by Bachmann [41] and Herzog [43] or Leonhardt [27], or in conjunction with several provisions in the OHBDC [29].

8/ FLEXURAL ANALYSIS

The trends of flexural investigations in partially prestressed beams as summarized by the report [2] fall into these two groups:

- The flexural performance is based on the homogeneous uncracked section where only the ultimate limit states are considered. Generally, methods are not suitable for use with limit states.
- The flexural performance is based on cracked section. Serviceability and ultimate limit states may be investigated.

In the following only the second group of methods will be discussed. The method proposed by S.E. Mustafa [34] provides formulation suitable for the investigation of rectangular and T-sections. This method does not incorporate all the requirements as previously discussed and may need introduction of additional information, however it definitely has potential.

The method by A.E. Naaman and A. Siriaksorn [33] is oriented toward the formulation used by ACI. A portion of the data in [33] may be used for limit states in the OHBDC [29] where the following features are of interest:

- the prestressing index parameter,
- design proposals of steel, and
- the formulation of full decompression level at service loads.

Crack control features and fatigue of steel are not original and are based on the information in [5, 22 and 45]. The method of A.H. Nilson [37] is providing the formulation of full decompression level, similar to that of previous authors.

In the following, the proposals by H. Bachmann [41] will be added. Data provided is a summary of Swiss experience and contains design features of a general nature. These data should be of interest to practicing engineers.

8.1/ Proposed Steps of Flexural Analysis

In Reference 41:

- First design step chosen is the selection of the prestressing index, where

$$X = M_v / M_q \quad (1)$$

M_v is the moment at decompression level equivalent to the moment due to prestress force after losses V_∞ plus the unfactored dead load moment (or part of it) and M_q is the unfactored moment due to a total of dead, live and dynamic loads.

For the majority of applications a prestressing index between $X = 0.6$ and $X = 0.7$ has been proposed. This choice should lead to minimal total requirements of steel.

The chosen prestressing index has distinct similarities with the proposal in Reference 33. Both expressions of prestress index may be used at will.

- In the second step, the designer shall establish the prestress force and the area of prestressing steel (A_v) from the condition of full decompression of the extreme tensile fibre of section.
- The third step serves to establish the area of unstressed reinforcement. This should be done using factored external loads with the criteria of ultimate states. The axial force component of prestress steel ($A_v \beta_{vs}$) is known from previous steps.

The ΣM equilibrium equation proposed for the ultimate moment is:

$$M_u = \gamma M_q = A_v \beta_{vs} Z_v + A_s \beta_{ss} Z_s \quad (2)$$

$$A_s = \frac{1}{\beta_{ss} Z_s} (M_u - A_v \beta_{vs} Z_v) \quad (3)$$

The original notations are as follows:

M_u = Ultimate moment

γM_q = Factored moments due to loads

A_v, A_s = The areas of prestressing and reinforcing steels, respectively.

β_{vs}, β_{ss} = Yield stress of prestressing and the yield of reinforcing steels, respectively.

Z_v, Z_s = Distances from centroids of prestressing and reinforcing steels to the centroids of the compression block.

Before proceeding to the fourth design step the following comments are in order.

The feature to be considered at ultimate when using the Equations 2 or 3 above is the need to incorporate the provision in Clause 9.11.2 of the OHBDC [29] which provides the requirements for under-reinforced beam. The advantage of Bachmann's Equation 3, however, is the ease with which the areas of both prestressed and non-stressed reinforcement can be obtained. Naaman's proposed equations [31] may be used to check the compatibility requirements at ultimate. In many cases when introducing the above-obtained steel ($A_v + A_s$) into Mustafa's method [34] as the area of "effective reinforcing", the common effective depth may be obtained from Equation 9 in Reference 43 (see Appendix).

- The fourth design step is assigned to investigate the fatigue of steel and/or provide the information to be used for crack control purposes. Any lineal elastic method known in reinforced concrete as "moment with compressive force" may be used to establish compressive stresses in concrete and tensile stresses in steel. Preference is given to the method proposed in Reference 34.

In the opinion of this writer, secondary preference is given to the equations for service loads proposed in Reference 33, which may be used with some success. Third preference is given to the equations by M. Herzog in [25] (Sections 4.2 and 4.3), or by Leonhardt in Sections 16 and 17, of [27].

Commenting further on the flexural investigations at serviceability level, the concrete should be assumed to be cracked below the neutral axis and the prestress force (V_{∞}) should assume full prestress losses due to effects of shrinkage, creep and relaxation. The location of the neutral axis for fatigue investigation in establishing stress range at the stress levels above full decompression in steel may differ from that for crack control check. During the investigation of crack control requirements, it should be remembered that the considerations by Thormählen [8]

or Trost [27] discussed in previous sections may apply for post-tensioned members. The effectiveness of bond in these sections shall be re-evaluated.

Bachmann and Mustafa do not emphasize the importance of accuracy when establishing the level of "full decompression." Both are of the opinion that relative stress values are more important (see Appendix).

The fatigue criteria should result from the stress range due to action of live load and dynamic component of live load only. In the bridge types with high ratio of dead to total load ($M_g/M_q = 0.6$) and equally high prestress index ($X = \frac{M_v}{M_g} > 0.6$) the full decompression effect may be about equal to the dead load; the fatigue will involve full live load effect. For a chosen ratio of $X > 0.7$, as in sections with limited prestressing, only part of live load and impact above full decompression level should be involved. As previously noted, actual stress ranges in steel of partially prestressed beams have been discussed in [2, 26, 27 and 41].

8.2/ Choice of Prestressing Index

In cases of non-composite beams, Bachmann suggests choosing the prestressing index in accordance with the following.

- For post-tensioned bonded sections the prestress index may vary between $X = 0.4$ to $X = 0.7$.
- In the beams with low dead to total moment ratios ($M_g/M_q = 0.4$) the prestressing index ($X = \frac{M_v}{M_q}$) should be higher than that ratio.
- When the prestress index chosen is, $X = 0.7$, the percentage of additional ordinary reinforcing in the effective section should be between 0.2% and 0.3%. The effective section in tension specified for crack control shall be used.
- This percentage of steel should be slightly increased when prestress index lies below the value of $X = 0.7$.

- The crack control requirements generally are satisfied using these recommendations. (See Table 1.)
- For prestress index $X < 0.4$ provisions for reinforced concrete should apply.
- To meet crack control requirements in pretensioned sections the percentage of ordinary reinforcing should generally be less than indicated above and often shall be only nominal.
- The method and the proposed considerations should not apply for the unbonded prestressed tendons. These sections exhibit increased cracking and lesser ultimate resistance.
- In post-tensioned beams with grouted tendons partial loss of bond has been indicated at the level of high loads. Leonhardt in [27] suggests improved crack pattern and ductility where loss of bond is compensated for with an addition of deformed reinforcing bars (see Appendix).

9/ SHEAR

The series of beams having various degrees of prestress, tested by several researchers [27] indicate positive effects of prestress on shear and torsion performance.

Tests by Bachmann [42] indicate that the inclined tendons may be to the disadvantage of partially prestressed beams, unless used with additional longitudinal reinforcing steel.

This non-prestressed reinforcing is suggested in both the full length bottom tension faces of all spans and in the top faces of continuous beams over the intermediate supports (top steel should reach beyond the points of contraflexure). In these zones reinforcing steel shall be placed along the tension outer face of beams in a manner typical for flexure reinforcing. Only then can prestressing tendons be lifted to accommodate moment requirements. Preferably, bottom steel should continue into supports and this steel should comply with bond provisions of ordinary reinforced beams.

The area as needed for this reinforcing at ultimate may be calculated from the vertical shear component (Q) at the supports and at (σ_s) first yield stress in steel.

$$A_s = Q/\sigma_s \quad (4)$$

Shear and bond problems arising from fatigue are indicated in Section 3, Reference 19, by Dr C.E. Ekberg Jr.

10/ CONCLUSIONS AND RECOMMENDATIONS

In conclusion, it is suggested that in partially prestressed concrete bridge design, the design logic of both reinforced and fully prestressed concrete should be adopted. The applicability of provisions should depend mainly on the choice of prestressing index as discussed. This opinion has been expressed in References 25, 41 and 54, and has been proven through tests [43].

The articles by Mustafa [34] and Bachmann [41] are considered to provide adequate formulation, adoptable for flexural analysis investigations in the majority of partially prestressed concrete bridge types. The flexure-oriented models of analysis should still be adjusted to comply with the provisions dictated by the limit states design as specified by the OHDBC [29].

The crack control equation still remains to be chosen and the discussions in Section 4 should be considered. Eventually, a new set of equations may be developed using basic suggestions by Thormählen [8] and others as indicated in [27].

The fatigue performance of anchorages and prestressing steels as applicable for Canadian market products should be certified by the manufacturers. These requirements should be based on the stress range concept as discussed in Section 5. In the meantime, for this purpose both Canadian information and that established in other countries should be evaluated and requalified by fatigue research engineers for use in partial prestress.

Further advance of the partial prestressing method in Ontario may be initiated by construction of a pilot bridge design. Substantial economical gains have been reported in some European countries -- mainly in Switzerland where prestressing methods are strongly advocated.

It would be most desirable to initiate some tentative clause in the OHDBC which would provide the guidelines for this type of design. Information provided by the Queen's University report [2] and that provided by this review, should be adequate to justify this action.

REFERENCES

- [1] "Causes and Control of Cracking in Concrete Reinforced with High Strength Steel Bars," Bulletin #479, University of Illinois, College of Engineering, Engineering Experiment Station (1965).
- [2] "State-of-the-Art Report on Partially Prestressed Concrete Bridge Structures," OJT & CRP Project #23156, Department of Civil Engineering, Queen's University at Kingston.
- [3] Abeles, P.W., "Full and Partially Prestressed Reinforced Concrete," ACI Journal, Vol. 41, No. 3, Jan. 1945, pp 181-216.
- [4] Rehm, G. and Martin, H., "Some of the Questions of Crack Control in Reinforced Concrete," Beton und Stahlbetonbau, Aug. 1968, pp. 175-182 (German).
- [5] Comité Euro-International du Béton (CEB) "Model Code for Concrete Structures," Final Draft, Paris, 1977.
- [6] Leonhardt, F., "Lectures on Concrete Structures -- Proof of Serviceability Criteria," Vol. 4, Springer-Verlag (1978) (German).
- [7] Leonhardt, F., "Crack Control in Concrete Structures," IABSE Surveys S-4/77, IABSE Periodica, Mar. 1977.
- [8] Thormählen, U., "Bond Effect on Crack Formation and Crack Control in Post-Tensioned Bonded Tendons of Partially Prestressed Structural Components." Beton und Stahlbetonbau, Oct. 1979, pp. 225-231 (German).
- [9] Trost, H., Cordes, H., and Thormählen, U., "Re: Proof of Crack Control According to DIN 4227," Beton und Stahlbetonbau, July 1980 pp. 169-173 (German).

- [10] "Control of Cracking in Concrete Structures," Report by ACI Committee 224, Concrete International, ACI, Vol. 2, No. 10, 1980, pp. 35-76.
- [11] Abeles, P.W., "Cracking and Bond Resistance in High Strength Reinforced Concrete Beams," ACI Journal, Vol. 63, No. 11, 1966, pp. 1265-1278.
- [12] ACI Committee 224, "Control of Cracking in Concrete-Structures," Concrete International, ACI, Vol. 2, No. 10, 1980, pp. 35-76.
- [13] Beeby, A.W., and Taylor, H.P.J., "Cracking in Partially Prestressed Members," Preliminary Publication, Sixth FIP Congress, Prague, June 1970, pp. 6-13.
- [14] Bennett, E.W., and Chandrasekhar, C.S., "Calculation of the Width of Cracks in Class 3 Prestressed Beams," Procs. Institution of Civil Engineers, Vol. 49, July 1971, pp. 333-346.
- [15] FIP-CEB Joint International Committee, "Recommendations for the Design and Construction of Concrete Structures," Cement and Concrete Association, June 1970, pp. 1-80.
- [16] Gergely, P., and Lutz, L.A., "Maximum Crack Widths in Reinforced Flexural Members," ACI Publications SP-20, American Concrete Institute, Detroit, 1968, pp. 86-117.
- [17] Goto, Y., "Cracks Formed in Concrete Around Deformed Tension Bars," ACI Journal, Vol. 68, No. 4, 1971.
- [18] Bennett, E.W., and Joynes, H.W., "Fatigue Resistance of Reinforcement in Partially Prestressed Beams," PCI Journal, Vol. 22, No. 2, March-April 1977, pp 78-89.
- [19] ACI Committee 215, "Consideration for Design of Concrete Structures Subjected to Fatigue Loading," ACI Journal, Vol. 71, No. 3, March 1974, pp. 97-121.

- [20] Price, K.M., and Edwards, A.D., "Fatigue Strength of Prestressed Concrete Flexural Members," Institution of Civil Engineers, Vol. 47, London, Oct. 1970, pp. 205-226.
- [21] Rao, S., and Babu, K.G., "Fatigue Behaviour of Partially Prestressed Concrete Members," Nonlinear Design of Concrete Structures, CSCE-ASCE-ACI-CEB International Symposium, University of Waterloo, Ontario, Canada, August 1979.
- [22] Hawkins, Neil M., "Fatigue Considerations for Reinforcements in Concrete Bridge Decks," ACI Journal, Feb. 1976, pp. 104-115.
- [23] Helgason, T., Hanson, J.M., Somes, N.F., Corley, W.G., and Hognes-tad, E., "Fatigue Strength of High Yield Reinforcing Bars," Final Report, NCHRP Projects 4-7 and 4-7(1), March 1975.
- [24] Rehm, G., Nürnberg, V., and Patzac, M., "Wedge and Clamp Type Pre-stress Anchors for High Strength Prestress Wires Under Dynamic Loads," Bauingenieur 52, 1977, pp. 327-398 (German).
- [25] Herzog, M., "Accumulative Fatigue Strength of Reinforced, Pre-stressed and Partially Prestressed Concrete," Die Bautechnik March 1977, pp. 73-80 (German).
- [26] Nürnberg, V., "Fatigue Behaviour of Prestressing Steels," Bauinge-nieur 56, 1981, pp. 311- 319 (German).
- [27] Leonhardt, I., "Lectures on Concrete Structures -- Prestressed Concrete," Vol. 5, Springer-Verlag, 1980 (German).
- [28] Abeles, P.W., "Partial Prestressing and its Suitability for Limit State Design," Structural Engineer, Vol. 49, No. 2, Feb. 1971.
- [29] "Ontario Highway Bridge Design Code", (Second Edition), Ministry of Transportation and Communications, Downsview, Ontario, Canada, 1983

- [30] Sozen, M.A., "Notes on Flexural Strength of Prestressed Concrete Beams," Prestressed Concrete Design Seminar, Chicago, pp. 25-27 January, 1971.
- [31] Naaman, A.E., "Ultimate Analysis of Prestressed and Partially Prestressed Sections by Strain Compatibility," PCI Journal, Vol. 22, No. 1, Jan.-Feb. 1977, pp. 32-51.
- [32] Naaman, A.E., and Siriaksorn, A., "Serviceability Based Design of Partially Prestressed Beams -- Part 1 Analysis," PCI Journal, Vol. 24, No. 2, March-April 1979, pp. 64-89.
- [33] Siriaksorn, A., and Naaman, A.E., "Serviceability Based Design of Partially Prestressed Beams -- Part II: Computerized Design and Parametric Evaluation," PCI Journal, Vol. 24, No. 3, May-June 1979, pp. 40-60.
- [34] Moustafa, S.E., "Design of Partially Prestressed Concrete Flexural Members," PCI Journal, Vol. 22, No. 3, May-June 1977, pp. 12-29.
- [35] Discussion of S.E. Moustafa's Paper "Design of Partially Prestressed Concrete Flexural Members," PCI Journal, Vol. 23, No. 3, May-June 1978, pp. 92-105.
- [36] Nawy, E.G., and Chiang, J.Y., "Serviceability Behaviour of Post-Tensioned Beams," PCI Journal, Feb. 1980.
- [37] Nilson, A.H., "Flexural Stresses After Cracking in Partially Prestressed Beams," PCI Journal, Vol. 21, No. 4, July-August 1976, pp. 72-81.
- [38] Thürlimann, B., "A Case for Partial Prestressing," Structural Concrete Symposium, University of Toronto, Toronto, Ont., Canada, May 1971, pp. 253-301.
- [39] Thompson, K.J. and Park, R., "Ductility of Prestressed and Partially Prestressed Concrete Beam Sections," PCI Journal, March-April 1980.

- [40] Leonhardt, F., "Recommendations for the Degree of Prestressing in Prestressed Concrete Structures," Presentation at a Joint Symposium, Organized by ACI, CEB, PCI and FIP held in Philadelphia 1976; Published in FIP Notes 69, July-August 1977 pp. 9-14.
- [41] Bachmann, H., "Experience in Switzerland with Partial Prestressing and the Questions in Design," Beton and Stahlbetonbau, Feb. 1980 (German).
- [42] Bachmann, H., "Testing of the Partially Prestressed Beams for the Effect of Tendon Inclination Upon the Shear Resistance," Zurich, Bauingenieur 51, 1976, pp. 251-258 (German).
- [43] Herzog, M., "The Flexural Ultimate Resistance of Reinforced and Partially Prestressed Concrete Based on the Test Results," Beton and Stahlbetonbau, March 1975 (German).
- [44] Beton-Kalender 1981, Vol. I and II, Wilhelm Ernst und John, Berlin.
- [45] ACI Committee 343 (Revised, formerly Committee 443), "Analysis and Design of Reinforced Concrete Bridge Structures," ACI Manual of Concrete Practice, Part 4, 1980, pp. 68-70.
- [46] Leonhardt, F., "Prestressed Concrete Design and Construction," (1964) Wilhelm Ernst und John.
- [47] Tadros, M.K., Ghalli, A., and Dilger, W.H., "Effect of Non-Prestressed Steel on Prestress Loss and Deflection," PCI Journal, V.22, No. 2. March-April, 1977, pp. 50-63.
- [48] Tadros, M.K., "Expedient Service Load Analysis of Cracked Prestressed Concrete Section," PCI Journal, V. 27, No. 6 Nov./Dec. 1982, pp. 86-111.

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APPENDIX A1/ Stress Transition in the Prestressing and Reinforcing Steel of Partially Prestressed Beams

Typical stress transition diagrams in the prestressing and reinforcing steels of the post-tensioned beam are shown in Figure A1.1. This diagram is intended to represent the partially prestressed beam, loaded with the gradually increasing load. The respective performance differences in the reinforced, partially, and fully prestressed beams will be indicated. The Stress Versus Load Transition Diagram illustrates three paths. The first, starting at the zero point of the co-ordinate system, represents reinforcing steel in a concrete beam; the second is reinforcing steel in a prestressed beam; and the third is the prestressing tendon.

At the start of the loading, the diagram shows the stress level in the prestressing steel as being equal to $(\sigma_{z,v_0} = 0.7 \beta_{0,2})$. At this time the reinforcing steel has been precompressed to a stress of $(\sigma_s = n\alpha_{bv})$. The diagram also indicates that due to shrinkage and creep losses $(\sigma_s + k)$, the tendon stress has been reduced accordingly. It will be assumed that due to the remaining prestress, the beam will lift up from the formwork -- an effect called "hogging". Due to this, some of the beam's own weight is assumed to be already "active" and results in stress change in the tendon, indicated as $(\sigma_{z,v+g})$. During this stress change in the tendon, the precompression stress in the reinforcing steel is indicated as decreasing to the level noted $(\sigma_s = n\alpha_{bv} = 0)$ and at the same time, the stress in the prestressing tendon has increased to the value $(\Delta\sigma_z = n\alpha_b)$, respectively. In the diagram, this stress buildup in the tendon $(\Delta\sigma_z)$ is very gradual and moderate. It may be assumed that the full decompression state will be reached in the reinforcing steel when the stress (α_b) , occurring at the reinforcing steel level in concrete will become equal and opposite to the (α_{bv}) concrete precompression stress. However it is customary in fully prestressed beams that both these stresses are calculated at the tendon level.

In partial prestressing, the intensity of stresses at the tendon level may be assumed to equal $(\Delta\sigma_z \cong \sigma_s = 100 \text{ MPa})$. This is considerably lower than the steel stress which could be anticipated due to the reinforced concrete beam's own weight.

The diagrams indicate the point at which precompression stress in the reinforcing steel (σ_s) and concrete stress (σ_b) at the prestressing steel level have reached the stress value indicated as ($\sigma_s \approx \sigma_b = 0$).

At this point, in fully prestressed beams, the load on the beam would be the full dead load plus the live load ($G + P$), respectively. In partial prestressing, however, depending upon the prestressing degree or index, the decompression stress in the steel may be reached either with the fraction of dead load (g) or with dead load plus a fraction of live load -- indicated in the diagram as ($g + \Delta p$). In analogy to H. Bachman [41], the prestressing index factor establishes the amount of external load moment on the beam at the time when full decompression is reached at the external tension fibre. The prestressing index (X) will be identified in partially prestressed beams in analogy with Reference 41 as:

$$X = \frac{\text{Decompression moment}}{\text{Total external moment}} = \frac{M_v}{\max M(G + P)} \quad (1.1)$$

Altering the point of zero stress reference appears to be logical, therefore, it may be suggested that (M_v) indicates the moment at which the full decompression in the concrete at the prestressing tendon level has been reached. Usually this level of stress may be identified with the prestressing force in the tendon after full prestress losses ($V_{z\infty}$). At the time of the decompression moment as indicated, the concrete is still uncracked. Shortly thereafter, however, the tensile strength of the concrete is exceeded and the first concrete crack is formed. The transfer of stress from concrete to steel ($\Delta\sigma_z$) has been noted in the diagram as stress jump. It is important to remember that the minimal amount of ordinary reinforcing provided in a fully prestressed beam should be such that the stress is not allowed to exceed the level specified in Clauses 9.8.2 and 9.13.2 of the OHBDC [29]. For partial prestressing, these provisions should be re-examined or waived. Alternative proposals to establish the decompression of concrete may be found in References 6, 37 and 38.

To clarify the importance of this stress jump, Figure A1.2 has been provided. It will be noted that total stress (σ_{eR}) at cracking load level may be of substantially different intensity where the intensity of this

this stress will be greater when a smaller reinforced steel ratio is provided. The indicated steel ratio (μ_z) must refer to the effective tension area of concrete (F_{bz}). Thus:

$$\mu_z = A_s \div F_{bz} \quad (1.2)$$

Returning to the stress jump in Figure A1.1, the stress ($\Delta\sigma_{z,s}$) is shared by both the prestressing and reinforcing steel, respectively.

It is indicated in the diagram that after the stress jump, the rate of stress increase assumes a much steeper angle than it did previously. For comparison purposes, in Figure A1.1, on the left lower corner, the stress change versus load is indicated for the ordinary reinforced beam. In partially and fully prestressed beams, the rate of this stress increase now assumes an equivalent ratio as indicated by the diagram. Generally, however, the stress "pick-up" in the reinforcing steel may be faster than in prestressing steel and this is mainly due to superior bonding conditions of the reinforcing bars. This feature is more outstanding in post-tensioned beams, where the bond may be related to the friction bond of the sheet and the quality of the grout.

This load sharing feature has been identified in Figure A1.1 by the shaded area on the prestressing tendon diagram. Several authors have indicated that the unequal rate of stress transfer between steels may result in non-linear strain distribution within the section or may even lead to bond failure in post-tensioned beam. To avoid this, Leonhardt repeatedly suggests [27] that additional reinforcing of the deformed bars be provided. The reinforcing is intended to "improve" the inferior bonding properties of the grouted tendon. Consequently, this additional reinforcement not only improves the bond properties of the beam, but also should prevent too rapid a climb of the neutral axis after the crack formation. It should therefore be expected that the cracks will be more evenly distributed and have smaller widths and depths.

Figure A1.1 assumes that the amounts of both the prestressing and reinforcing steels have been selected in such a manner that both steels will reach their respective tensile limits simultaneously. Figure A1.1 also indicates that the yield stress (β_s) in the reinforcing steel and the

0.2% strain offset -- proof stress ($\beta_{z,0.2}$) -- in the prestressing steel are reached at the time when the full factored ultimate dead and live loads $\gamma(G + P)$ were acting on the beam. Although details of the compatibility requirements specified by the OHBD Code provisions are not identified in Figure A1.1, they are referred to in Clauses 9.11.2 and 9.13, respectively [29].

In the conclusion, it is noted that contrary to the stress-load transition indicated for the reinforced concrete beam, the rate of stress increase in partially prestressed beams is not proportional to the rate of the applied load as has been illustrated in the diagram. Consequently, the section strength may not be correctly identified at the level of service loads.

At the present time, it is customary to assume some point on the diagram beyond that of full decompression ($\alpha_b = \alpha_s = 0$) which is to be identified with the load levels assigned for the check of serviceability criteria. In the serviceability check of fatigue, deformation and crack control criteria, it has generally been agreed to assume the "cracked" concrete section. Earlier, in Section 8 of this report, a proposal was made to investigate the serviceability of the partially prestressed beam under simultaneous action of the prestress force including the appropriate reduction due to creep, shrinkage, and the appropriate prestress reducing portion of the dead load where applicable (as indicated in Figure A1.1).

To this effect, several alternative proposals have been made in the different codes [27, 29 and 44]. Among these, the load factor ranging from 0.7 to 1.35 was proposed only for applied loads during the check of cracking criteria. Furthermore, it is surprising that only a few of the codes specify the temperature restraints to be considered.

The many alternatives in the provisions of the various codes are indicative of the diversity of opinion that exists among engineers. Some of these features still remain to be resolved for the OHBDC.

Many engineers suggest that additional effort should be made to establish more accurate values of the prestress losses when checking serviceability

fatigue criteria. It is apparent, therefore, that there may be a need to separate cracking and fatigue analysis using different levels of accuracy.

In conclusion, it still may be indicated that after the full decompression level indicated in Figure A1.1, the additional stresses in the prestressing and reinforcing steel would be proportionally smaller when the higher prestressing index has been chosen.

The transition diagrams of steel strains will be discussed next in order to further illustrate the implementation of the serviceability and ultimate limit states criteria in partially prestressed beams.

NOTE: -- The above section was based on information provided in References 6, 8, 27, 29, 30, 31 and 41.

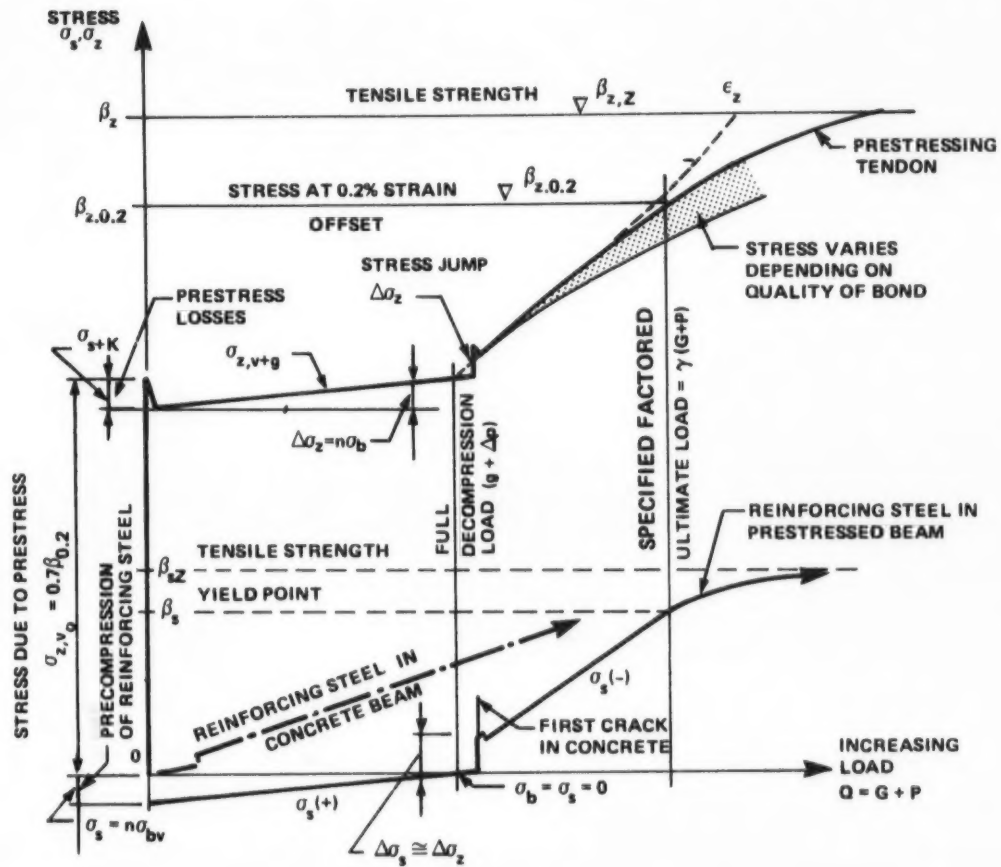


Figure A1.1/ Stress Versus Load-Transition Diagram .
 During the gradual increase of load Q , stress changes in the reinforcing steel and the prestressing steel of partially prestressed beam are shown. (σ_s, σ_z)
 (According to Ref. 27 Chapter 6)

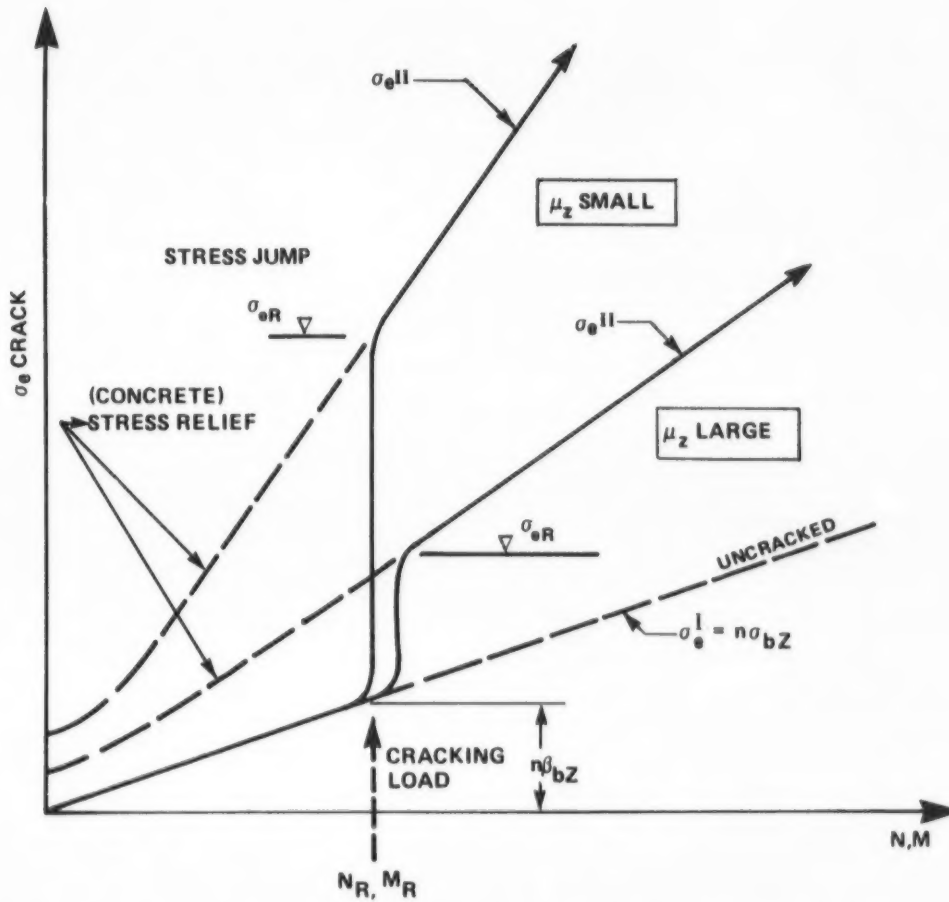


Figure A1.2/ Diagram of Stress Jump in the Steel After the Crack Formation in a Reinforced Concrete Tension Member.

Stress jump $\Delta\sigma_{eR} = \sigma_{eR} - \sigma_e^I = \sigma_{eR} - n\beta_{bz}$

Magnitude of the steel stress (σ_{eR}) varies with differing tensile strengths of concrete (β_{bz}) and the reinforcing steel ratios (μ_z) respectively. [According to F. Leonhardt Ref. 6 Chapter 2]

APPENDIX A2/ Transition of Strains in Partially Prestressed Beams

The transition of strains and stresses increase up to fracture in reinforcing and prestressing steels in a prestressed concrete beam is indicated in Figure A2.1.

Two transition levels are to be discussed. First, the strain and stress levels which are to be permitted at the ultimate limit states shall be defined. Then the strain in the prestressing steel and reinforcing steel at the level of full decompression will be discussed. The diagram shown was originally intended mainly for the case of a fully prestressed beam, but it may also be applied to partially prestressed beams.

With ideal bonding conditions, the total strain in prestressing steel ϵ_{su} may be assumed at ultimate [29] as:

$$\epsilon_{su} = \epsilon_{se} + \epsilon_{ce} + \epsilon_{sa} \quad (2.1)$$

where: ϵ_{sa} is the increase in strain in the prestressing steel compatible with the $\epsilon_u = 0.003$, compressive strain in concrete (Clause C9.11.2);

$\epsilon_{se} + \epsilon_{ce}$ is the strain in the steel plus the strain in concrete induced by the prestressing force ($V_{\infty}^{(0)}$), and equal to strain:

$$\epsilon_{z, V_{\infty}}^{(0)} = \epsilon_{se} + \epsilon_{ce}$$

where: $\epsilon_{z, V_{\infty}}^{(0)} = \epsilon_{z, V_0}^{(0)} + \frac{1}{E_z} (\sigma_{s+k} - n \sigma_{b, s+k})$

f_{se} is the effective stress in the prestressing steel after losses $\equiv \sigma_{V_{\infty}}^{(0)}$, where the prefix (o) denotes that the stress relates to the prestressing force in the prestressing bed ($V_{\infty}^{(0)}$).

According to the OHBDC [29], the limiting stress in the prestressing steel at ultimate should be assumed as (f_{ps}) . This stress is slightly higher than that indicated in the diagram as $(\beta_{z,0.2})$. Figure A2.1 indicates limiting stress for the reinforcing steel as (σ_s) . This intensity of stress usually is taken as equal to the yield stress for the given type or reinforcing. It has been stipulated, in the diagram, that theoretically both steel types are reaching the above-specified ultimate stress levels simultaneously. Although this condition is not necessarily true in practice the strain limit designated in the diagram as $(\epsilon_{z,u})$ will correspond to both these stress limits. This strain should be assumed to be equal to or less than $(\phi \epsilon_{su})$, when the reinforcing steel yields first as usually is the case.

$$\epsilon_{z,u} \leq \phi \epsilon_{su} \leq \epsilon_q \quad (2.2)$$

Figure A2.1 indicates that maximum strain permitted in the reinforcing steel should not exceed the value of $\epsilon_q = 0.5\%$. This strain limitation is intended to impose some control on the width and depth of cracks forming at the high load level close to ultimate. This feature has been introduced from provisions of DIN 1045 and DIN 4227, but is not intended to change the existing compatibility provisions in the OHBDC [29]. Thus at $\epsilon_q = 0.5\%$ in steel, the maximum usable strain in concrete should not exceed $\epsilon_u = 0.3\%$. The requirement of Clause 8.7.4.1.1 should be re-examined.

It may now be inferred that the strain (ϵ_q) as shown in Figure A2.1 should be applied to check the position of the stress at full decompression indicated as $(\sigma_{v\infty}^{(0)})$. This suggestion would be justified if strain and stress for each steel had resulted from equal bonding conditions. In post-tensioned bonded tendons, however, this condition is difficult to accomplish in reality. Therefore, to avoid the design inaccuracies, the strain $\epsilon_q = 0.5\%$ should be looked upon mainly for reference.

During the design of a partially prestressed beam, the following expression, which has been derived from Equation 2.1, should be considered in order to estimate the point on the diagram of full decompression. In reference to the point of zero stress and strain:

$$\phi \epsilon_{su} \equiv \epsilon_{zu} = \epsilon_{z,v\infty}^{(0)} + \epsilon_q \quad (2.2)$$

where:

$\epsilon_{z,v\infty}^{(0)}$ is the strain in the prestressing steel after prestress losses plus the load due to "hogging" effect or own weight as may be applicable. Often, when a lower bound prestressing index has been chosen, this condition of strain would relate to a load less than its own weight. Strain also shall include strain transferred from concrete at cracking.

$\epsilon_q \equiv \epsilon_{sa}$ is the maximum strain in the reinforcing and prestressing steel due to the applied loads after full decompression of concrete at the tendon level.

Stress at full decompression as indicated in Figure A2.1 is identical to a reference point on the strain diagram which was suggested earlier in Appendix A1 discussing Figure A1.1. After this point when investigating the serviceability criteria, "cracked section" should be assumed. This condition of strain in steel may be accounted for if stress levels at cracking (σ_{eR}) as indicated in Figure A1.2 are known. (The level of the stress reference at the decompression level may be assumed to be at the level of reinforcing steel.)

Continuing discussion on decompression, the strains in the prestressing and the non-prestressing steel are affected by concrete creep. Subsequently, strains in the prestressing steels are reduced and the compressive strains in the non-prestressing steels are increased.

In consequence of the above-discussed stress transfer at cracking and stress redistribution due to creep, it is estimated that on the average

the non-prestressing steel should gain some 100 MPa of precompression and the prestressing steel should retain some of the 300 MPa tensions.

Thus the earlier-suggested choice of force ($V_{z,\infty}$) for moment (M_V) to establish the point of decompression was fully justified and is on the safe side.

NOTE: -- The above section was based on the information provided in References 27, 29, 30, 31, 32, 34, 35, 37, 38, 41, 47 and 48.

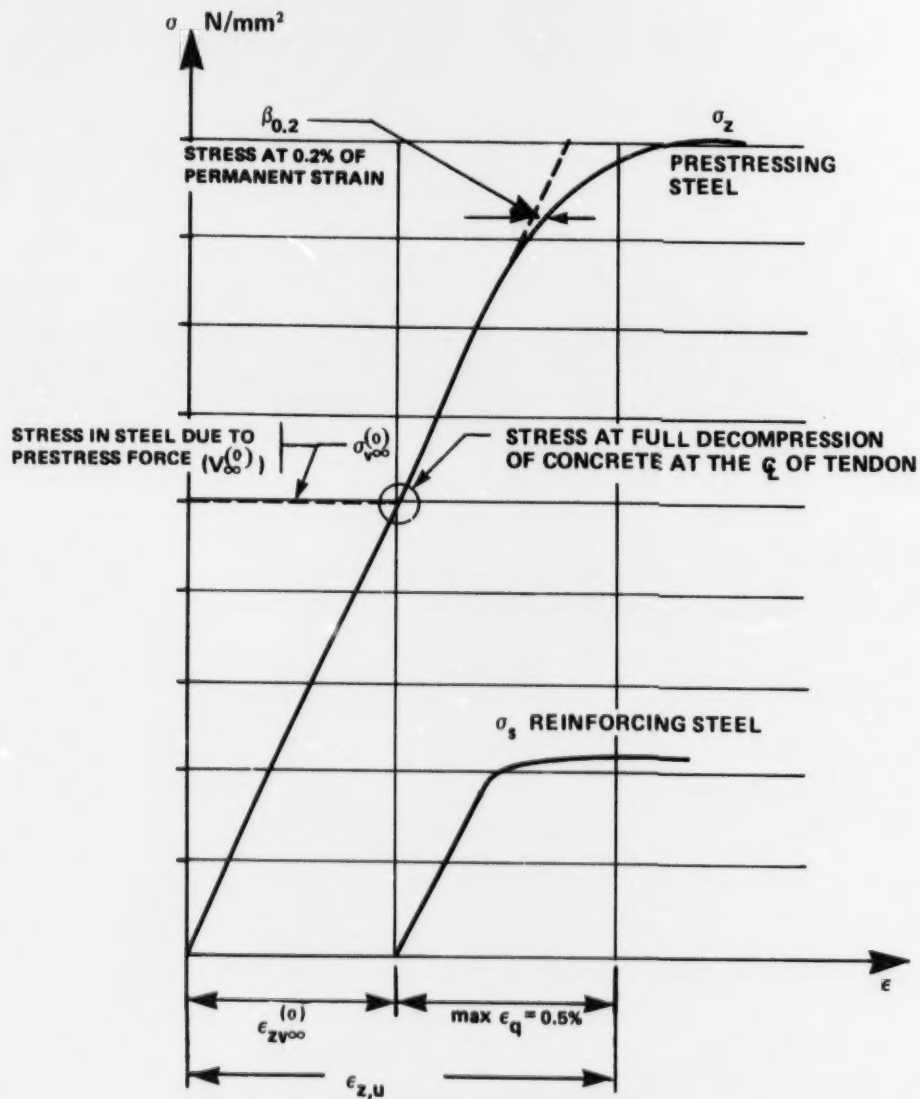


Figure A2.1/ Assumed Transition of Stress and Strain in the Prestressed Concrete Beam.

The strain ($\epsilon_{z,u}$) signifies the limits permissible in partially prestressed beam. The strain ($\epsilon_{zv\infty}$) indicates the full precompression of reinforcing steel due to prestress force. [According to F. Leonhardt Ref. 27 Chapter 18]

APPENDIX A3/ Crack Formation and Investigation Proposals for Serviceability Criteria in Partially Prestressed Concrete Beams

A3.1/ Crack Formation and Investigation Proposals for Crack Control

Figure A3.1 shows a stress-strain model for reinforcing steel in an unstressed concrete beam. This model indicates the stage when stable concrete crack has been already formed. At this stage it is assumed the cracks have reached the minimal possible spacing indicated as (a).

The stress at cracking (σ_{eR}) which was shown in Figure A1.2, has been expressed in Figure A3.1 as steel stress (σ_e^{II}) and strain (ϵ_e^{II}), respectively. The equation for the crack width (w_m), based on the crack spacing (a), may, according to Leonhardt [6], be expressed as:

$$w_m = v_o \cdot \epsilon_e^{II} + (a_m - v_o) \cdot \epsilon_m \quad (3.1)$$

Stress (σ_e^{II}) and strain (ϵ_e^{II}) in Figure A3.1 are at the centre of concrete cracks. It has been proposed that distributions of uniform intensity be assumed for stress and strain. The length interval of (v_o) indicates the "disturbed" bond condition between steel and concrete. In the adjacent interval, indicated as ($a - v_o$), strain ϵ_e^{II} has been reduced to the mean value of (ϵ_m). The reduction of this steel strain was indicated as strain ($\Delta\epsilon_e$) and stress ($\Delta\sigma_e$), respectively. Both of these reductions are due to the action of concrete between cracks.

Figure A3.2 has been presented to further illustrate this concrete action. This idealized stress-strain diagram for a tension member of composite concrete-steel has been proposed by Rostasy [6, Chapter 2]. In the diagram, the dotted line which represents the action of reinforcing steel alone is indicated as "naked" steel alone. Then the composite action of both steel and concrete is indicated "from tests" as the solid line. The hatched area between the dotted and the solid lines indicates the participating effect of concrete. It will be noted that the majority

of notations provided in this figure are analogical to those used in Figure A3.1. But in addition it is assumed in Figure A3.2 that strain (ϵ_{eR1}) is the steel strain at the time when the first crack was forming; when cracks have reached the mean interval of stable spacing, steel strain (ϵ_e^{II}) in Figure A3.2 indicates the respective strain reductions (a result of concrete action). These are noted here as ($\max \Delta\epsilon_e$) and ($\Delta\epsilon_e$).

The mean value of strain indicated as (ϵ_m) at the time of stable crack spacing, and in accordance with References 4 and 6, may be expressed as:

$$\epsilon_m = \epsilon_e^{II} - \Delta\epsilon_e = \frac{1}{E_e} (\sigma_e^{II} - \Delta\sigma_e) \quad (3.2)$$

In the expression ($\Delta\sigma_e$) is strain relief due to concrete between cracks. In the diagram, the reference of strain (ϵ_e^{II}) is made to the point of zero strain. In partial prestressing this is the point of decompression for the reinforcing steel, as previously discussed and shown on Figure A1.1.

The above expression has been further modified by H. Falkner [6], where the steel stress during the first cracking (σ_{eR1}) has been reintroduced into Equation 3.2.

He assumed the following:

$$\epsilon_e^{II} = \frac{\sigma_e^{II}}{E_e}; \quad \sigma_{eR1} = \frac{\beta_{bz}}{\mu_z}; \quad \Delta\sigma_e = k_5 \frac{\sigma_{eR1}^2}{\sigma_e^{II}}$$

Thus, substituting into Equation 3.2:

$$\epsilon_m = \epsilon_e^{II} - \Delta\epsilon_e = \epsilon_e^{II} - \frac{\sigma_{eR1}^2}{\sigma_e^{II}} \cdot \frac{k_5}{E_e} = \epsilon_e^{II} \left[1 - k_5 \left(\frac{\sigma_{eR1}}{\sigma_e^{II}} \right)^2 \right] \quad (3.3)$$

In this equation, the reduction in steel stress due to concrete action between the cracks is to be multiplied by the factor (k_5). This factor is intended to reflect possible changes of steel stress ($\Delta\sigma_e$).

As suggested in Reference 6, the factor should vary from $k_5 = 0.8$ to $k_5 = 0.4$. The variation is intended to reflect first the fatigue, then the effects at the sustained load, respectively.

As may be observed from Figure A1.2, the steel stress at cracking (σ_{eR1}) is the product of stress ($\Delta\sigma_{eR}$) in steel and the tensile strength of concrete. The concrete area for this relation is assumed to be different for tension concrete members and flexure members, respectively. A relation for (σ_{eR}) has been established in Chapter 2 of Reference 6. For the reinforced concrete members, the stress jump ($\Delta\sigma_{eR}$) indicated in Figure A1.2 can vary between values of 20 to 500 MPa as indicated in Figure 2.8a and 2.8b of Reference 6 or Figure 6 and 7 of Reference 7. The stress may be lower than 500 MPa in the case of partial prestressing.

In partially prestressed beams, further modifications of Equation 3.1 may be introduced. It has been indicated by Leonhardt [6] that the bond stress distribution between the cracks in partially prestressed concrete is different to that in unstressed members. The major differences suggested are the lower bond stress intensity (β_{T1}) and the possible disappearance of the "disturbed bond" interval (V_0). These changes are indicated in Figure A3.3. In the extreme case where ($V_0 = 0$), it may be assumed that the crack width Equation 3.1 will be reduced to read:

$$w_m = a_m \cdot \epsilon_m \quad (3.4)$$

In the above equation, (a_m) is the mean value of stable crack spacing.

The original equation by Rehm and Martin [4] has been further modified by Thormahlen [8] and the modified equation for crack spacing reads:

$$a_m = k_1 \cdot f(C_b, S) + k_2 \cdot k_3 \cdot \frac{F_{bz}}{\Sigma U_s + C \Sigma U_z} \quad (3.5)$$

In this expression, the first term indicates the function combining the effect of concrete cover of the reinforcing (C_b) and the bar spacing (S), respectively. The second term of the original expression $\left(\frac{F_{bz}}{\sum S}\right)$ has been expanded to accommodate partially prestressed beams. In this term the (F_{bz}) represents the "effective area" of concrete which, after cracking, was transferring the tensile stress from concrete to steel. The indicated notations ($\sum S$ and $\sum Z$) denote the sums of the parameters for the reinforcing and the prestressing steels, respectively. The indicated factor (C) is intended to modify bond value in post-tensioned bonded steel. These factors are given in Table A3.2-1. Using Equation 3.5 for pretensioned section, it shall be assumed that $C = 1$.

We could introduce Equations 3.3 and 3.5 into Equation 3.4. Because the expression should represent the critical crack width (W_{CR}) in partially prestressed beams, the factor (k_4) should be introduced to reflect the scatter of the test data. Then,

$$W_{CR} = W_m \cdot k_4 = k_4 \cdot a_m \cdot \epsilon_m \quad (3.6)$$

This expression has the equivalent form to the 1978, CEB 4.2.2 recommendation provided in [12] or [2]. The selection of all factors (k) may be made using References 4, 5, 6 and 8. It is contended that further simplification should still be possible. For this reason, conclusive proposals should be made following completion of further investigation.

A3.2/ Applications for the Investigation of Crack Control and Other Criteria

A3.2.1/ Crack Control

The information provided in the following section is to be applicable for the analysis of crack control criteria in post-tensioned partially prestressed concrete beams.

The bond reduction factor (C) to be used in Equation 3.5 has been adopted from the investigations by Thormählen [8] and similar investigations by

Trost [27]. These researchers relate this factor to the slip $\Delta = 0.1$ mm during the pull-out tests of the post-tensioned grouted tendons. The grout values during the tests were varied from 30 to 50 MPa strength. The following relation was used to establish factor (C):

$$C = \frac{\tau_{bz}}{\tau_{bs}} = \frac{\text{bond stress for prestressed tendon } (\Delta = 0.1 \text{ mm})}{\text{bond stress for deformed reinforcing } (\Delta = 0.1 \text{ mm})}$$

(It will be assumed that directly bonded tendons have general properties of bond equal to those of the deformed reinforcing bars, $C = 1.0$.) Following Table A3.2-1 applies to post-tensioning tendons grouted within the corrugated sheets.

Table A3.2-1/

$C = 0.2$ -- Smooth bars or wire tendons.
$C = 0.4$ -- Wire strands, oval shape deformed wires or wires with indentations.
$C = 0.6$ -- Ribbed or threaded bars.

The use of factor (C) will be demonstrated in the following application.

- Often the area of the prestressing tendon and the area of the reinforcing steel may be combined into one area unit (A_e). For post-tensioned beams:

$$A_e = A_s + C \cdot A_z \quad (3.7)$$

This combined steel area unit shall be located within the "effective" area of concrete in tension (F_{bz}), References 6 and 29. The factor (C) is provided in Table A3.2-1.

- In order to express the individual effective depths of the reinforcing steel (d_s) and the effective depth of the prestressing tendon (d_p)

as one common effective depth (d), the following approximation may be used:

$$d = \frac{d_s \cdot A_s + d_p \cdot C \cdot A_z}{A_s + C \cdot A_z} \quad (3.8)$$

(A more elaborate equation is seldom needed for crack control investigations.)

The steel parameters (U_s , U_z) for Equation 3.5 may be found as follows. In accordance with Chapter 5 of [27], the parameter (U_z) for the prestressing bar, wire or the strand tendon when bonded within the corrugated sheet is obtained as:

$$U_z = 1.6 \cdot \pi \cdot \sqrt{A_z} \quad (3.9)$$

The parameter for bar (U_s) or tendon (U_z) in direct bond is:

$$U_s = U_z = \pi \cdot d = \pi \cdot 1.13 \cdot \sqrt{A_{s,z}} \quad (3.10)$$

A3.2.2/ Applications for Other Criteria

As mentioned previously, the expressions in Section A3.2 were derived for post-tensioned partially prestressed beams. Other situations will be listed where the expressions may apply. However, engineering judgement should be used during the following applications:

- In the case of pretensioned members, Equations 3.7 and 3.8 are applicable provided the bond reduction factor $C = 1.0$ will be used.
- Equations 3.7, 3.8, 3.9 and 3.10 may be applied, not only for the investigation of crack control but also, for the investigation of fatigue. In the absence of more precise information, the factor $C = 1.0$ should apply for both post-tensioned and pretensioned beams.

- For the investigation of bond, the (C) factor from Table A3.2-1 may be suggested only in post-tensioned beams. Otherwise, $C = 1.0$ shall be used.

NOTE: -- The above section (A3) was based on information provided in References 4, 5, 6, 7, 8, 9, 15, 27, 43, and 44.

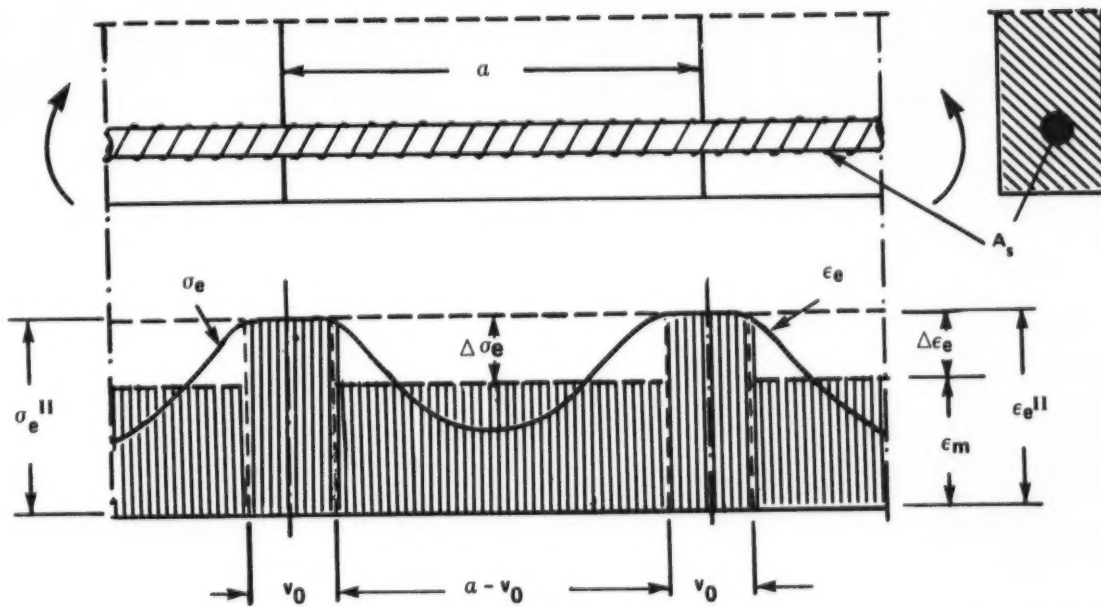


Figure A3.1/ Model of Stress and Strain in the Reinforcing Steel At and Between Two Stable Cracks in a Concrete Beam. The strain interval noted as (v_0) indicating the disturbed bond condition between steel and the concrete. (Ref.6 Chapter 2).

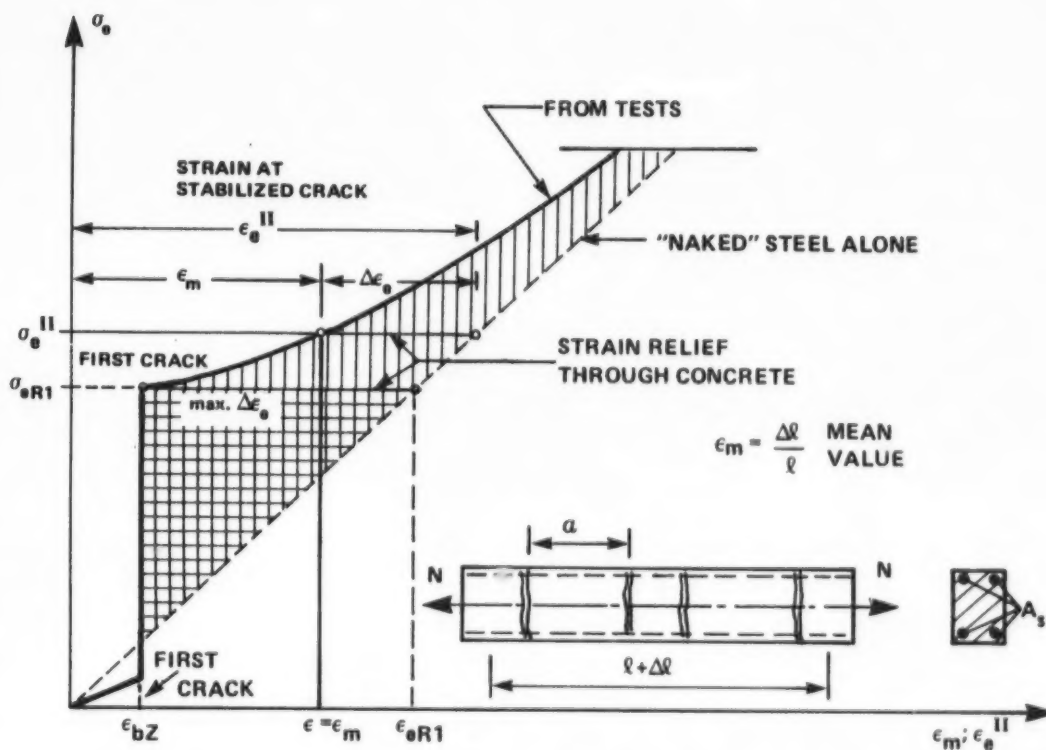


Figure A3.2/ Idealized Stress-Strain ($\sigma-\epsilon$) Diagram for a Tension Reinforced Concrete Member (by Rostásy).

Diagram indicates composite action of steel and concrete and the assumed performance for reinforcing steel alone. Strain relief through concrete between the cracks is indicated. (Ref. 6 Chapter 2)

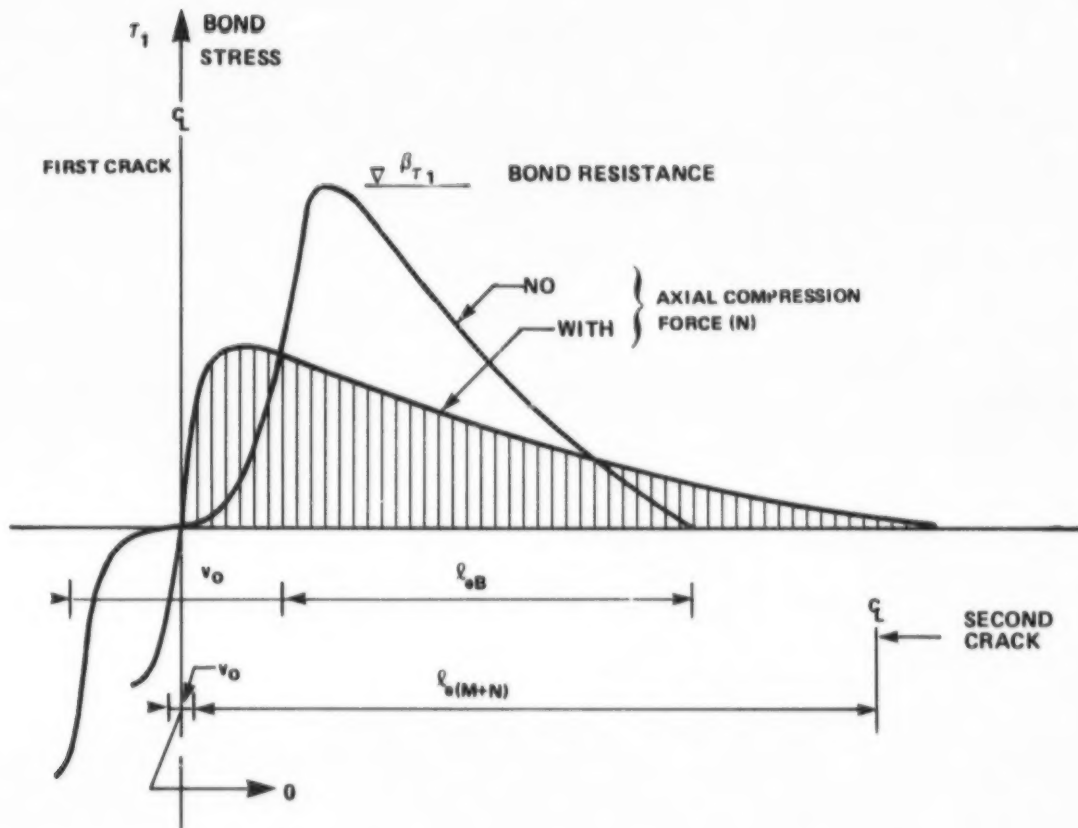


Figure A3.3/ Model Indicates Different Bond Stress Levels in Unstressed and Partially Prestressed Concrete Member, Respectively. Smaller stress jump ($\Delta\sigma_{eR}$) in steel and flatter bond stress transfer diagram is suggested for the prestressed member. [According to F. Leonhardt Ref. 6 Chapter 2]

APPENDIX A4/ Available Fatigue Criteria in Reinforced and Prestressed Concrete -- With a Comparison of Extracts From Various Codes

The fatigue criteria have been discussed in Section 5 of this review. The information provided was intended to inform the reader about the design and test data related to steel and concrete materials subjected to fatigue loading. The information in Section 5 was based on the reference material from American, Swiss and German sources as was indicated.

Extracts from the ACI Committee 343, from OHBDC and DIN 1045, DIN 488 and DIN 50100 are included in this Appendix. In particular, the extracts from the ACI and DIN codes are provided in order to illustrate basic code differences of the fatigue requirement stipulated for reinforced and fully prestressed concrete sections in America and Germany.

Before any of these provisions or requirements could be applicable to the design of partially prestressed bridges in the OHBDC, the information presented must be subject to evaluation and official approval. The main differences between the ACI and DIN requirements concern mainly the allowable stress ranges of the reinforcing and the prestressing steels. They may be followed by comparing Appendix Sections A4.1, A4.2 and A4.3, respectively.

There is a need for internationally acceptable and uniform design procedures. Although the pending recommendations for the OHBDC are left open, it is strongly suggested that the selections of these recommendations be made within the CEB/FIP Model [5].

NOTE: -- The above section is based on information provided in References 6, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 41, 44, 45, and 46.

Appendix A4.1/ Extract From ACI Committee 343 (formerly 443) [45]

8.3/ Fatigue of Materials

8.3.1/ Reinforcing Bars

8.3.1.1/

For straight hot-rolled bars with no welds and with no stress raisers more severe than deformations meeting the requirements of AASHTO reinforcing bar specifications, the range between a maximum tension stress and minimum stress caused by live load plus impact at service load should not exceed:

$$F_{sr} = 21 - 0.33 f_{min} + 8(r/h) *$$

where:

F_{sr} = stress range calculated as algebraic difference between maximum stress and minimum stress when tension is positive and compression is negative, ksi (MPa)*

f_{min} = algebraic minimum stress level, tension positive, compression negative, ksi (MPa)*

r/h = ratio of base radius to height of rolled on transverse deformation; where actual value is not known, 0.3 is recommended.

Greater stress ranges may be used if determined by fatigue tests on similar bars. These recommendations are based on Reference 23.

* The equation expressed in SI units [29] is:

$$F_{sr} = 145 - 0.33 f_{min} + 55(r/h) \quad (\text{MPa})$$

8.3.1.2/

For bars containing welds conforming to requirements of American Welding Society "Reinforcing Steel Welding Code" AWS D12.1-75 range between a maximum tension stress and minimum stress caused by live load plus impact at service load in the reinforcing element in the vicinity of the weld should not exceed that allowed by Section 8.3.1.1 nor 18 ksi (124 MPa).

8.3.1.3/

Bends and tack welds in principal reinforcement should not be used in regions of high stress range.

8.3.2/ Concrete

The compressive stress in the concrete due to live load plus impact at service load should not exceed $0.5f'_c$ at sections where stress reversals occur. No concrete fatigue failures have been known to occur but the points of stress reversal would be the most critical.

8.3.3/ Prestressing Steel

The stress range in prestressed reinforcement that may be imposed on minimum stress levels up to 60% of the tensile strength should not exceed the following:

Strand and bars	0.10 f_{pu}	**
Wire	0.12 f_{pu}	

(For additional information, see References 19 and 22.)

**This leads to stress range limit of the order of:

Strands	169 MPa
Wires	190 to 210 MPa
Threadbars	103 MPa
Threadbar Couplers and Anchorages	103 MPa

Appendix A4.2/ Extract From the OHBDC [29]

REFERENCE PUBLICATIONS

Sections 8 and 9 make reference to the following publications:

CSA Standards	CAN3-A23.1-M77	Concrete Materials and Methods of Concrete Construction
	CAN3-A23.2-M77	Methods of Test for Concrete
	CAN3-A23.3-M77	Code for the Design of Concrete Structures for Buildings
	G30.12-M77	Billet Steel Bars for Concrete Reinforcement
	G30.16-M77	Weldable Low-Alloy Steel Deformed Bars for Concrete Reinforcement
	A251-1971	Qualification Code for Manufacturers of Architectural and Structural Precast Concrete
	W186-1970	Welding of Reinforcing Bars in Reinforced Concrete Construction
	G30.3-1972(R1979)	Cold-Drawn Steel Wire for Concrete Reinforcement
	G279-M1982	Steel for Prestressed Concrete Tendons

Appendix A4.3/ Extract From DIN 1045, DIN 488 and DIN 50100

Table A4.3.1/ Approved Stress Ranges $2\sigma_A$ for German Reinforcing Steels

Steel Type		BSt 220/340 GU	BSt 220/340 RU	BSt 420/500 RU	BSt 420/500 RK
Mark		IG	IR	III U	III K
Range of diameter (mm)		5 - 28	6 - 40	6 - 28	6 - 28
Yield or proof stress β_s or $\beta_{0.2}$ - Min.		220	220	420	420
Tensile Strength β_t - Min.		340	340	500	500
Fatigue Endurance Strength For Stress Range $2\sigma_A = \sigma_0 - \sigma_u$	Straight Bars	180	-	230	230
	Bent Bars D = 15 de	180	-	200	200

All stresses are indicated in (MPa)

LEGEND: G = Smooth; R = Ribbed; U = Untreated; K = Cold Formed

FROM: DIN 1045 and DIN 488, Vol. 1 pp. 194 [44]

Table A4.3.2/ Fatigue Strength (Endurance) of Prestressing Steels
-- Stress Range From Tests

Fatigue endurance limit of 2×10^6 stress cycles of load repetitions at upper stress limit of $\sigma_0 = 0.9 \beta_s$ or $0.8 \beta_z$ (yield and tensile strength, respectively).

Type of Prestress Steel	Stress Range $2\sigma_A$
Smooth Wire Smooth Bar	240 N/mm ²
Ribbed Wire Deformed Wire Strand	200 N/mm ²
Threaded Wire Threaded Bar	190 N/mm ²

...According to F. Leonhardt [27], Chapter 4.

(The above stress range values are based on tests done in accordance with DIN 50100.)

Table A4.3.3/ Fatigue Strength (Endurance) of the Prestress Anchorage Types Indicated -- Stress Range ($2\sigma_A$)

Fatigue endurance limit of 2×10^6 stress cycles of load repetitions in bonded tendons. Minimum stress range $2\sigma_A = 80 \text{ kN/mm}^2$ is specified for the anchorages of the unbonded tendons.

STRESS RANGES FOR THE ANCHORAGES OF TENDON TYPES

Type of Anchorage	$2\sigma_A \text{ (N/mm}^2\text{)}$
Bars ϕ 26 mm, St 590/885 to St 885/980	
Screw Thread, Metric Standard Nut	~ 65
Special Thread, Dywidag Wedgenut	to 140
Screw Threaded Socket	109
Screw Thread Gewi-Bar Anchor	75 to 90
Wires ϕ 5 to 8 mm, St 1375/1570 to St 1570/1770	
Wedges, According to Shape, Normal	110 to 140
Wedges, with Prevented Friction Corrosion	160
Drawn Sockets, Standard	~ 120
Drawn Sockets, with Aluminum Insert	~ 230
Loops, Leoba-Tendon	180
Loops, 7-wire, Strand in Concrete	250
Buttonheads, BBRV	180
New Shape BBRV-Dyna	260
Bond Anchorages	200

...According to F. Leonhardt [27], Chapter 10.

(The above values of stress ranges are based on tests done in accordance with DIN 50100.)

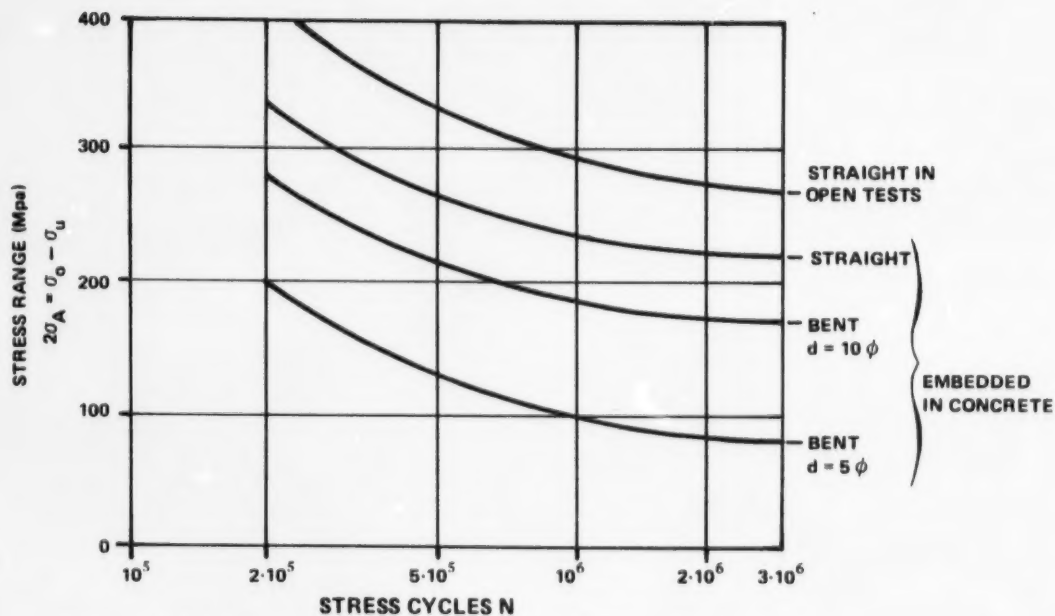


Figure A4.3.1/ Fatigue Strength (Endurance) of Reinforcing Bars .

Fatigue endurance limits in excess of 2×10^6 stress cycles for load repetitions at upper stress limit of $\sigma_o = 0.8\beta_{0.2}$ was tested. The curves indicate permitted stress ranges $2\sigma_A$ for deformed (ribbed) bar types on 1045-BStIR, BStIII V and BStIII K respectively. [Wöhler Curves by G. Rehm and H. Wascheidt, from [6]Chapter 3]